On the electromagnetic interactions of Dirac and Weyl particles

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Introduction (1)

This presentation focuses on the electromagnetic interactions of Dirac and Weyl particles, showing that under special conditions they can exhibit extraordinary behavior.

Specifically, we have shown that all Weyl particles, and under certain conditions Dirac particles, can exist in the same quantum state under a wide variety of electromagnetic 4-potentials and fields, infinite in number, which are explicitly calculated.

Introduction (2)

We have also discussed a set of particularly interesting solutions to the Weyl equations, showing that Weyl particles can exist in localized states even in the absence of electromagnetic fields.

Furthermore, the localization of the Weyl particles can be easily adjusted using simple electric fields.

Based on these results we have proposed a novel device for controlling the flow of information at a rate of up to 100 Petabits per second using Weyl fermions.

Introduction (3)

These results are expected to find interesting applications in several fields of science and technology, such as nanophotonics, nanoelectronics, laser physics, solid state physics, etc., providing new pathways for further development in these fields, both in theory and applications.

Our results are summarized in the following articles (1)

[1] A. I. Kechriniotis, C. A. Tsonos, K. K. Delibasis and G. N. Tsigaridas, On the connection between the solutions to the Dirac and Weyl equations and the corresponding electromagnetic 4-potentials, arXiv:1208.2546 [math-ph], Commun. Theor. Phys. 72 (2020) 045201, DOI: 10.1088/1572-9494/ab690e

[2] G. N. Tsigaridas, A. I. Kechriniotis, C. A. Tsonos and K. K. Delibasis, Degenerate solutions to the Dirac equation for massive particles and their applications in quantum tunneling, arXiv:2010.09844 [quant-ph], Phys. Scr. 96 (2021) 065507, DOI: 10.1088/1402-4896/abf304

[3] G. N. Tsigaridas, A. I. Kechriniotis, C. A. Tsonos and K. K. Delibasis, Degenerate solutions to the massless Dirac and Weyl equations and a proposed method for controlling the quantum state of Weyl particles, arXiv:2010.09846 [quant-ph], Chin. J. Phys. 77, 2324-2332(2022) DOI: 10.1016/j.cjph.2022.04.010

Our results are summarized in the following articles (2)

[4] G. N. Tsigaridas, A. I. Kechriniotis, C. A. Tsonos and K. K. Delibasis, On the localization properties of Weyl particles, arXiv:2205.11251 [quant-ph], Ann. Phys. (Berlin) 2200437 (2022) DOI: 10.1002/andp.202200437

[5] G. N. Tsigaridas, A. I. Kechriniotis, C. A. Tsonos and K. K. Delibasis, Degenerate wave-like solutions to the Dirac equation for massive particles, arXiv:2209.10933 [quant-ph], EPL 142 (2023) 50001, DOI: 10.1209/0295-5075/acd473

[6] G. N. Tsigaridas, A. I. Kechriniotis, C. A. Tsonos and K. K. Delibasis, A general method for obtaining degenerate solutions to the Dirac and Weyl equations and a discussion on the experimental detection of degenerate states, arXiv:2210.02003 [quant-ph], Ann. Phys. (Berlin) 2200647 (2023) DOI: 10.1002/andp.202200647

[7] G. N. Tsigaridas, A. I. Kechriniotis, C. A. Tsonos and K. K. Delibasis, A proposed device for controlling the flow of information based on Weyl fermions, arXiv:2307.06489 [quant-ph], Sensors 24 (2024) 3361, DOI: 10.3390/s24113361

Dirac's equation

$$i\gamma^{\mu}\partial_{\mu}\Psi + a_{\mu}\gamma^{\mu}\Psi - m\Psi = 0$$

$$\gamma^{0} = \begin{pmatrix} \sigma^{0} & 0 \\ 0 & -\sigma^{0} \end{pmatrix} \quad \gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ -\sigma^{\mu} & 0 \end{pmatrix}$$

$$\sigma^{0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma^{1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^{2} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^{3} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$a_{\mu} = qA_{\mu} \qquad \hbar = c = 1$$

Inverse problem

Given the wave function Ψ, what can we say about the electromagnetic 4-potential A_μ which is connected to Ψ by Dirac's equation?

Is A_μ uniquely determined, and if not, what is the extent to which it is arbitrary?

C. J. Eliezer, A Consistency Condition for Electron Wave Functions, Camb. Philos. Soc. Trans. 54 (1958) 247.

Answer (1)

If $\Psi^{\dagger}\gamma\,\Psi \neq 0$ then Ψ corresponds to a unique 4-potential defined by the formula

$$a_{\mu} = \frac{i}{2}\frac{\bar{\Psi}\gamma^{5}\gamma^{\mu}\not{\partial}\Psi - \bar{\Psi}\gamma^{5}\overleftarrow{\not{\partial}}\gamma^{\mu}\Psi}{\bar{\Psi}\gamma^{5}\Psi}$$

$$\gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3} \qquad \gamma = \gamma^{0} + \gamma^{0}\gamma^{5}$$

$$\bar{\Psi} = \Psi^{\dagger}\gamma^{0} \qquad \not{\partial} = \gamma^{\mu}\partial_{\mu}$$

$$\bar{\Psi}\gamma^{\nu}\overleftarrow{\not{\partial}}\gamma^{\mu}\Psi = \overline{\not{\partial}\Psi}\,\gamma^{\nu}\gamma^{\mu}\Psi$$

H. S. Booth , G. Legg and P. D. Jarvis, Algebraic solution for the vector potential in the Dirac equation, J. Phys. A: Math. Gen. 34 (2001) 5667

Answer (2) – Theorem 5.4 in [1]

If $\Psi^{\dagger}\gamma\,\Psi = 0$ and $\Psi^{T}\gamma^{2}\,\Psi \neq 0$ then Ψ corresponds to an infinite number of real 4-potentials of the form

$$b_{\mu} = a_{\mu} + f\,\theta_{\mu}$$

where *f* is an arbitrary real function of the spatial coordinates and time *and*

$$\left(\theta_0,\theta_1,\theta_2,\theta_3\right) = \left(1, -\frac{\Psi^{T}\gamma^{0}\gamma^{1}\gamma^{2}\Psi}{\Psi^{T}\gamma^{2}\Psi}, -\frac{\Psi^{T}\gamma^{0}\Psi}{\Psi^{T}\gamma^{2}\Psi}, \frac{\Psi^{T}\gamma^{0}\gamma^{2}\gamma^{3}\Psi}{\Psi^{T}\gamma^{2}\Psi}\right)$$

Remark*:* At least two of the above 4-potentials correspond to different electromagnetic fields

The above solutions are called **degenerate**

Answer (3)

If $\Psi^\dagger \gamma\, \Psi = 0$ and $\Psi^T \gamma^2\, \Psi = 0$ then Ψ can be written in the form

$$\Psi = \begin{pmatrix} \psi \\ \psi \end{pmatrix} \quad \text{or} \quad \Psi = \begin{pmatrix} \psi \\ -\psi \end{pmatrix}$$

where ψ is solution to the Weyl equations.

Weyl equations

$$a_\mu \sigma^\mu \Psi = -i\sigma^\mu \partial_\mu \Psi$$

corresponding to massless particles with spin parallel to their propagation direction (positive helicity)

$$2a_0 \sigma^0 \Psi - a_\mu \sigma^\mu \Psi = -\left(2i\sigma^0 \partial_0 \Psi - i\sigma^\mu \partial_\mu \Psi\right)$$

corresponding to massless particles with spin anti-parallel to their propagation direction (negative helicity)

On the degeneracy of the solutions to the Weyl equations – Theorem 3.1 in [1]

All the solutions to the Weyl equations are degenerate, corresponding to an infinite number of real 4-potentials, given by the formulae

$$b_\mu = a_\mu + f\varphi_\mu$$

where *f* is an arbitrary real function of the spatial coordinates and time *and*

$$(\varphi_0, \varphi_1, \varphi_2, \varphi_3) = \left(1, \pm\frac{\Psi^\dagger\sigma^1\Psi}{\Psi^\dagger\Psi}, \pm\frac{\Psi^\dagger\sigma^2\Psi}{\Psi^\dagger\Psi}, \pm\frac{\Psi^\dagger\sigma^3\Psi}{\Psi^\dagger\Psi}\right)$$

Here the plus and minus sign correspond to the cases of negative and positive helicity respectively.

On the degeneracy of free Dirac particles

It is proven that the wavefunction Ψ of a free Dirac particle is degenerate if and only if the particle is massless.

In this case, the wavefunction Ψ corresponds to an infinite number of 4-potentials of the form

$$\left(a_0, a_1, a_2, a_3\right) = \left(1, -\sin\theta\cos\varphi, -\sin\theta\sin\varphi, -\cos\theta\right) s\left(\mathbf{r}, t\right)$$

where $s(\mathbf{r},\mathrm{t})$ is an arbitrary real function of the spatial coordinates and time, and θ, φ are the angles defining the propagation direction of the particle in spherical coordinates.

Electromagnetic fields conserving the state of massless free Dirac particles

Setting the electric potential $\varphi(\mathbf{r},t) = a_0/q$

the magnetic vector potential $\mathbf{A}(\mathbf{r},t) = -(1/q)\,(a_1, a_2, a_3)$

and using the relations

$$\mathbf{E}(\mathbf{r},t) = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t} \qquad \mathbf{B}(\mathbf{r},t) = \nabla\times\mathbf{A}$$

we obtain the electromagnetic fields conserving the state of massless free Dirac particles

$$\mathbf{E}(\mathbf{r},t) = -\nabla s_q - \frac{\partial s_q}{\partial t}\left(\sin\theta\cos\varphi\,\mathbf{i} + \sin\theta\sin\varphi\,\mathbf{j} + \cos\theta\,\mathbf{k}\right)$$

$$\mathbf{B}(\mathbf{r},t) = \left(\cos\theta\frac{\partial s_q}{\partial y} - \sin\theta\sin\varphi\frac{\partial s_q}{\partial z}\right)\mathbf{i} - \left(\cos\theta\frac{\partial s_q}{\partial x} - \sin\theta\cos\varphi\frac{\partial s_q}{\partial z}\right)\mathbf{j} +$$

$$\sin\theta\left(\sin\varphi\frac{\partial s_q}{\partial x} - \cos\varphi\frac{\partial s_q}{\partial y}\right)\mathbf{k},\ \text{where } s_q(\mathbf{r},t) = s(\mathbf{r},t)/q$$

The violation of Ohm's law

If the arbitrary function *g* depends only on time, the magnetic field becomes zero, and the electric field takes the simpler form

$$\mathbf{E}(\mathbf{r},t) = -\frac{ds_q}{dt}(\sin\theta\cos\varphi\mathbf{i} + \sin\theta\sin\varphi\mathbf{j} + \cos\theta\mathbf{k})$$

The same result is also true for free Weyl particles.

Consequently, the state of the particles will not be affected in the presence of an electric field, of arbitrary magnitude and time dependence, applied along their direction of motion.

This practically means that the electric current transferred by charged particles in degenerate states will not change if a voltage, of arbitrary magnitude and time dependence, is applied along the direction of motion of the particles, contrary to what is expected in the framework of classical physics.

On the interaction of massless Dirac and Weyl particles with electromagnetic waves (1)

Assuming that the arbitrary function $g(\mathbf{r},t)$ is of the form

$$s_q(\mathbf{r},t) = -E_{w1}\cos\left[k_w(z-t)+\delta_{w1}\right]x - E_{w2}\cos\left[k_w(z-t)+\delta_{w2}\right]y$$

the electric and magnetic field conserving the state of free massless Dirac or Weyl particles moving along the *+z* direction become

$$\mathbf{E}(\mathbf{r},t) = E_{w1}\cos\left[k_w(z-t)+\delta_{w1}\right]\mathbf{i} + E_{w2}\cos\left[k_w(z-t)+\delta_{w2}\right]\mathbf{j}$$

$$\mathbf{B}(\mathbf{r},t) = -E_{w2}\cos\left[k_w(z-t)+\delta_{w2}\right]\mathbf{i} + E_{w1}\cos\left[k_w(z-t)+\delta_{w1}\right]\mathbf{j}$$

corresponding to a plane electromagnetic wave propagating along the *+z* direction.

On the interaction of massless Dirac and Weyl particles with electromagnetic waves (2)

Here, $E_{w1}, E_{w2}, \delta_{w1}, \delta_{w2}$ are arbitrary real constants corresponding to the amplitude and phase of the x and y component of the electric field of the wave respectively, and k_w is a real parameter corresponding to the wavenumber.

Thus, the state of free massless Dirac or Weyl particles is not affected by a plane electromagnetic wave, e.g. a laser beam, of arbitrary polarization, propagating along the direction of motion of the particles.

Consequently, particles and waves propagate without interacting with each other, which obviously is not true for “ordinary” charged particles.

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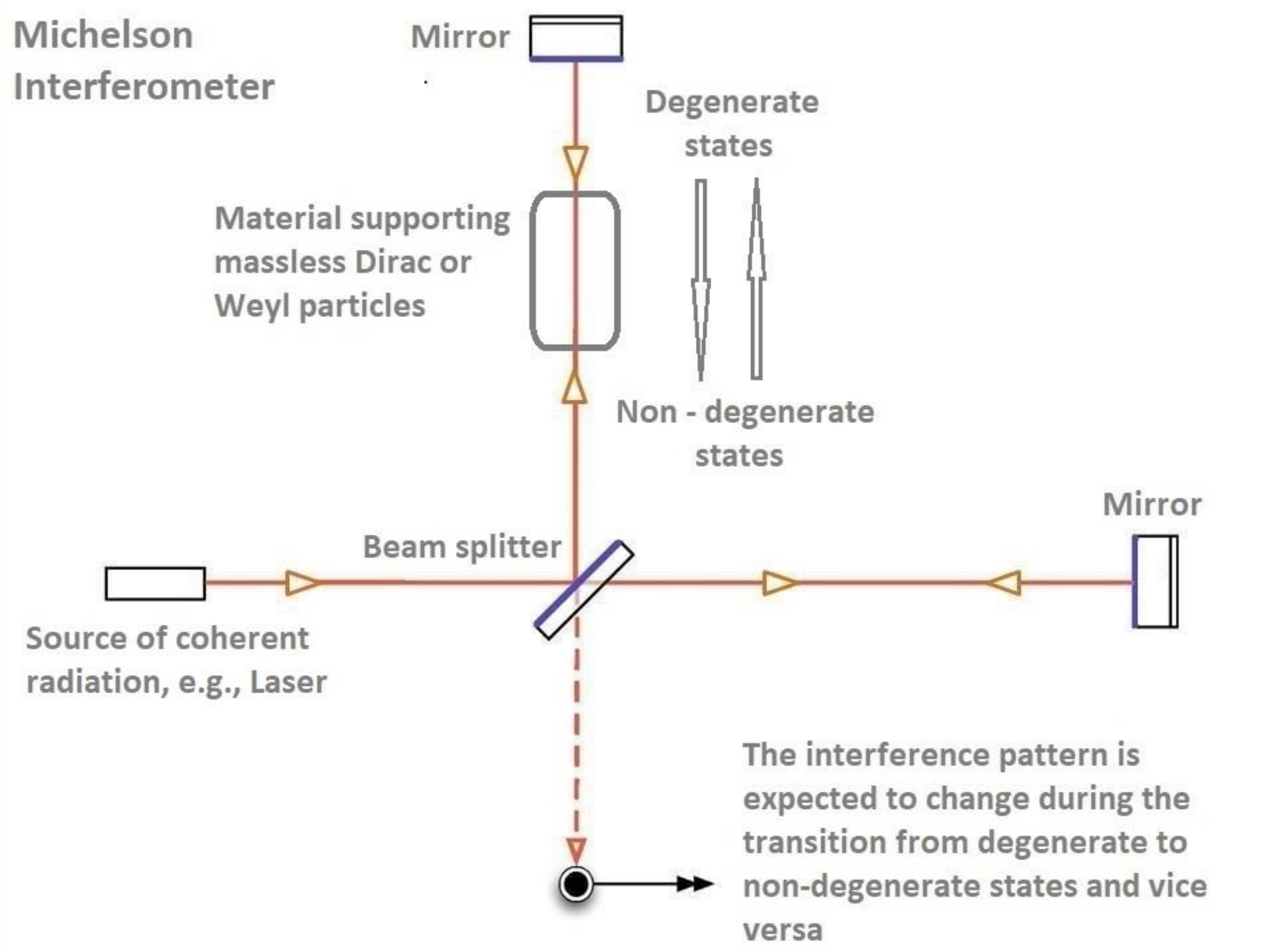
A proposed method for experimentally detecting the transition between degenerate and non – degenerate states



Regarding the practical applications of these results (1)

All the above results indicate that free massless Dirac and Weyl particles have the property to interact with electromagnetic fields in an extraordinary way, like no other known charged particle, behaving as charged photons.

The problem is that massless Dirac and Weyl particles do not exist in nature as standalone entities.

Consequently, are all these remarkable results useless?

No, because it has been shown recently that massless Dirac and Weyl particles can exist in certain materials as "quasi-particles", corresponding to collective excitations of the electrons.

K. S. Novoselov et al, Two-dimensional gas of massless Dirac fermions in graphene, Nature 438 (2005) 197.
D. Ciudad, Weyl fermions: Massless yet real, Nat. Mater. 14 (2015) 863.
H. -H. Lai, S. E. Grefe, S. Paschen and Q. Si, Weyl-Kondo semimetal in heavy-fermion systems, P. Natl. Acad. Sci. USA 115 (2018) 93.

Regarding the practical applications of these results (2)

Thus, the property of massless Dirac and Weyl particles to be able to exist in the same quantum state under a wide variety of different electromagnetic fields is expected to provide new opportunities regarding the development of novel devices and applications involving materials supporting massless quasi-particles, such as graphene, Weyl semi-metals, etc.

Solutions to the Weyl equations with particular interest

It can be shown that all spinors of the form

$$\psi = \begin{pmatrix} \cos\left(\dfrac{\theta(t)}{2}\right) \\ e^{i\varphi(t)}\sin\left(\dfrac{\theta(t)}{2}\right) \end{pmatrix} \exp\left[ih(x,y,z,t)\right]$$

where *h* is an arbitrary real function of the spatial coordinates and time, are solutions to the Weyl equation

$$i\sigma^{\mu}\partial_{\mu}\Psi + a_{\mu}\sigma^{\mu}\Psi = 0$$

corresponding to particles with positive helicity or antiparticles with negative helicity.

Solutions to the Weyl equations with particular interest (2)

Similarly, all spinors of the form

$$\psi' = \begin{pmatrix} -\sin\left(\dfrac{\theta(t)}{2}\right) \\ e^{i\varphi(t)}\cos\left(\dfrac{\theta(t)}{2}\right) \end{pmatrix} \exp\left[ih(x,y,z,t)\right]$$

are solutions to the Weyl equation

$$i\sigma^{\mu}\partial_{\mu}\Psi - 2i\sigma^{0}\partial_{0}\Psi + a_{\mu}\sigma^{\mu}\Psi - 2a_{0}\sigma^{0}\Psi = 0$$

corresponding to particles with negative helicity or antiparticles with positive helicity.

Electromagnetic 4-potentials corresponding to these solutions (1)

The electromagnetic 4-potentials corresponding to the spinors ψ are of the form

$$b_\mu = a_\mu + s\kappa_\mu$$

where

$$\left(a_0, a_1, a_2, a_3\right) = \left(\frac{\partial h}{\partial t} + \frac{1}{2}\frac{d\varphi}{dt}, \frac{\partial h}{\partial x} + \frac{1}{2}\sin\varphi\frac{d\theta}{dt}, \frac{\partial h}{\partial y} - \frac{1}{2}\cos\varphi\frac{d\theta}{dt}, \frac{\partial h}{\partial z} - \frac{1}{2}\frac{d\varphi}{dt}\right)$$

and

$$\left(\kappa_0, \kappa_1, \kappa_2, \kappa_3\right) = \left(1, -\sin\theta\cos\varphi, -\sin\theta\sin\varphi, -\cos\theta\right)$$

As usually, *s* is an arbitrary real function of the spatial coordinates and time.

Electromagnetic 4-potentials corresponding to these solutions (2)

Similarly, the electromagnetic 4-potentials corresponding to the spinors ψ′ are of the form

$$b'_\mu = a'_\mu + s\kappa_\mu$$

where

$$\left(a'_0, a'_1, a'_2, a'_3\right) = \left(\frac{\partial h}{\partial t} + \frac{1}{2}\frac{d\varphi}{dt}, \frac{\partial h}{\partial x} - \frac{1}{2}\sin\varphi\frac{d\theta}{dt}, \frac{\partial h}{\partial y} + \frac{1}{2}\cos\varphi\frac{d\theta}{dt}, \frac{\partial h}{\partial z} + \frac{1}{2}\frac{d\varphi}{dt}\right)$$

and

$$\left(\kappa_0, \kappa_1, \kappa_2, \kappa_3\right) = \left(1, -\sin\theta\cos\varphi, -\sin\theta\sin\varphi, -\cos\theta\right)$$

Electromagnetic fields corresponding to these solutions (1)

The electromagnetic fields corresponding to the 4-potentials

$$\left(a_0, a_1, a_2, a_3\right) = \left(\frac{\partial h}{\partial t} + \frac{1}{2}\frac{d\varphi}{dt}, \frac{\partial h}{\partial x} + \frac{1}{2}\sin\varphi\frac{d\theta}{dt}, \frac{\partial h}{\partial y} - \frac{1}{2}\cos\varphi\frac{d\theta}{dt}, \frac{\partial h}{\partial z} - \frac{1}{2}\frac{d\varphi}{dt}\right)$$

are given by the formulae

$$\mathbf{E} = \frac{1}{2q}\left(\cos\varphi\frac{d\theta}{dt}\frac{d\varphi}{dt} + \sin\varphi\frac{d^2\theta}{dt^2}\right)\mathbf{i} + \frac{1}{2q}\left(\sin\varphi\frac{d\theta}{dt}\frac{d\varphi}{dt} - \cos\varphi\frac{d^2\theta}{dt^2}\right)\mathbf{j} - \frac{1}{2q}\frac{d^2\varphi}{dt^2}\mathbf{k}$$

$$\mathbf{B} = \mathbf{0}$$

Electromagnetic fields corresponding to these solutions (2)

Similarly, the electromagnetic fields corresponding to the 4-potentials

$$\left(a_0', a_1', a_2', a_3'\right) = \left(\frac{\partial h}{\partial t} + \frac{1}{2}\frac{d\varphi}{dt}, \frac{\partial h}{\partial x} - \frac{1}{2}\sin\varphi\frac{d\theta}{dt}, \frac{\partial h}{\partial y} + \frac{1}{2}\cos\varphi\frac{d\theta}{dt}, \frac{\partial h}{\partial z} + \frac{1}{2}\frac{d\varphi}{dt} \right)$$

are given by the formulae

$$\begin{aligned} \mathbf{E}' &= -\frac{1}{2q}\left(\cos\varphi\frac{d\theta}{dt}\frac{d\varphi}{dt} + \sin\varphi\frac{d^2\theta}{dt^2} \right)\mathbf{i} - \frac{1}{2q}\left(\sin\varphi\frac{d\theta}{dt}\frac{d\varphi}{dt} - \cos\varphi\frac{d^2\theta}{dt^2} \right)\mathbf{j} + \frac{1}{2q}\frac{d^2\varphi}{dt^2}\mathbf{k} \\ &= -\mathbf{E} \\ \mathbf{B}' &= \mathbf{0} \end{aligned}$$

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On the property of Weyl particles to exist at different quantum states in zero electromagnetic field (1)

It can be easily confirmed that if the following condition holds

$$\frac{d^2\theta}{dt^2}=\frac{d^2\varphi}{dt^2}=\frac{d\theta}{dt}\frac{d\varphi}{dt}=0$$

the electromagnetic field becomes zero.

On the property of Weyl particles to exist at different quantum states in zero electromagnetic field (2)

Consequently, Weyl particles in zero electromagnetic field exhibit one of the following behaviors:

- move as free particles, assuming that

$$d\theta/dt = d\varphi/dt = 0$$

- exist in a localized bounded state in the case that

$$d\theta/dt = \omega_1, \;\; d\varphi/dt = 0$$

- exist in an intermediate state, bound on the x-y plane, and free along the z-axis, corresponding to

$$d\theta/dt = 0, \;\; d\varphi/dt = \omega_2$$

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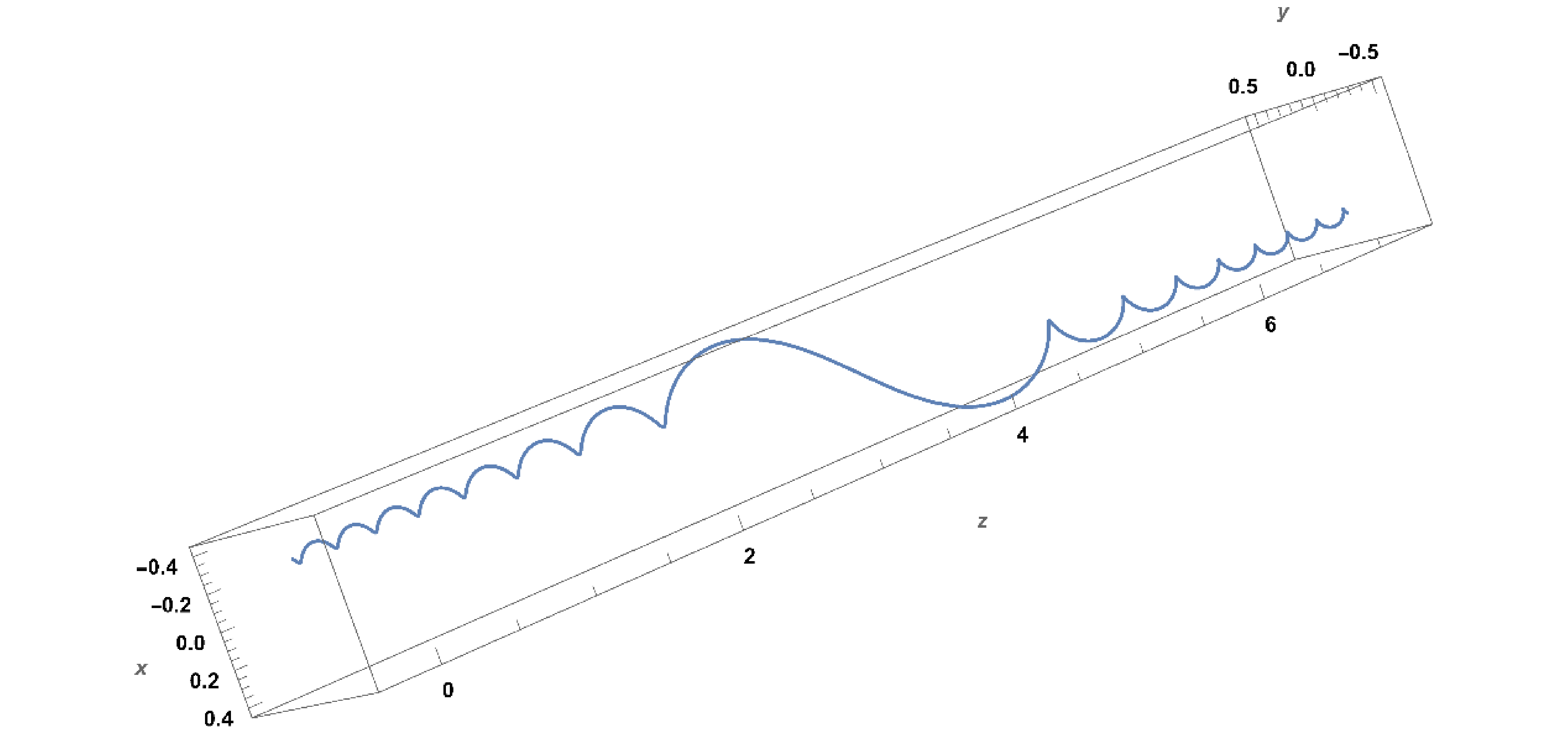
The trajectory of a classical particle with the same velocity as the Weyl one for $\theta(t)=\pi/4$ and $\varphi(t)=20t-2t^2$, corresponding to a constant electric field $\mathbf{E}=(2/q)\mathbf{k}$, applied for $t\in[0,10]$



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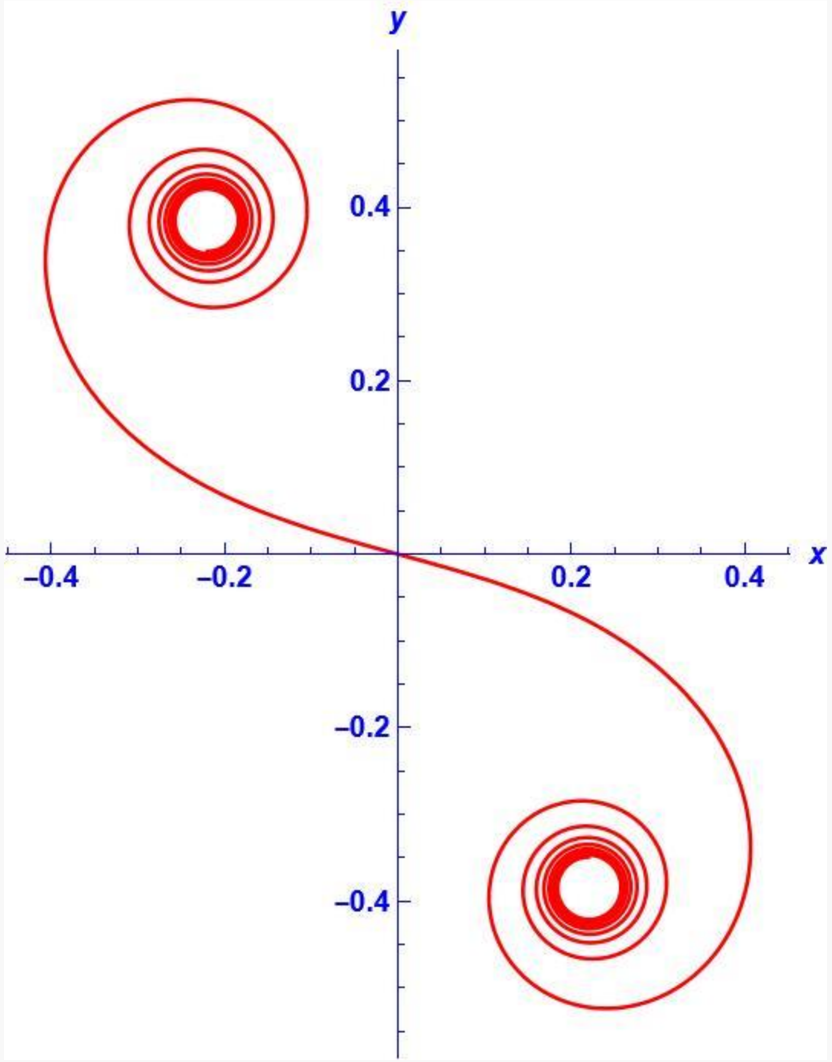
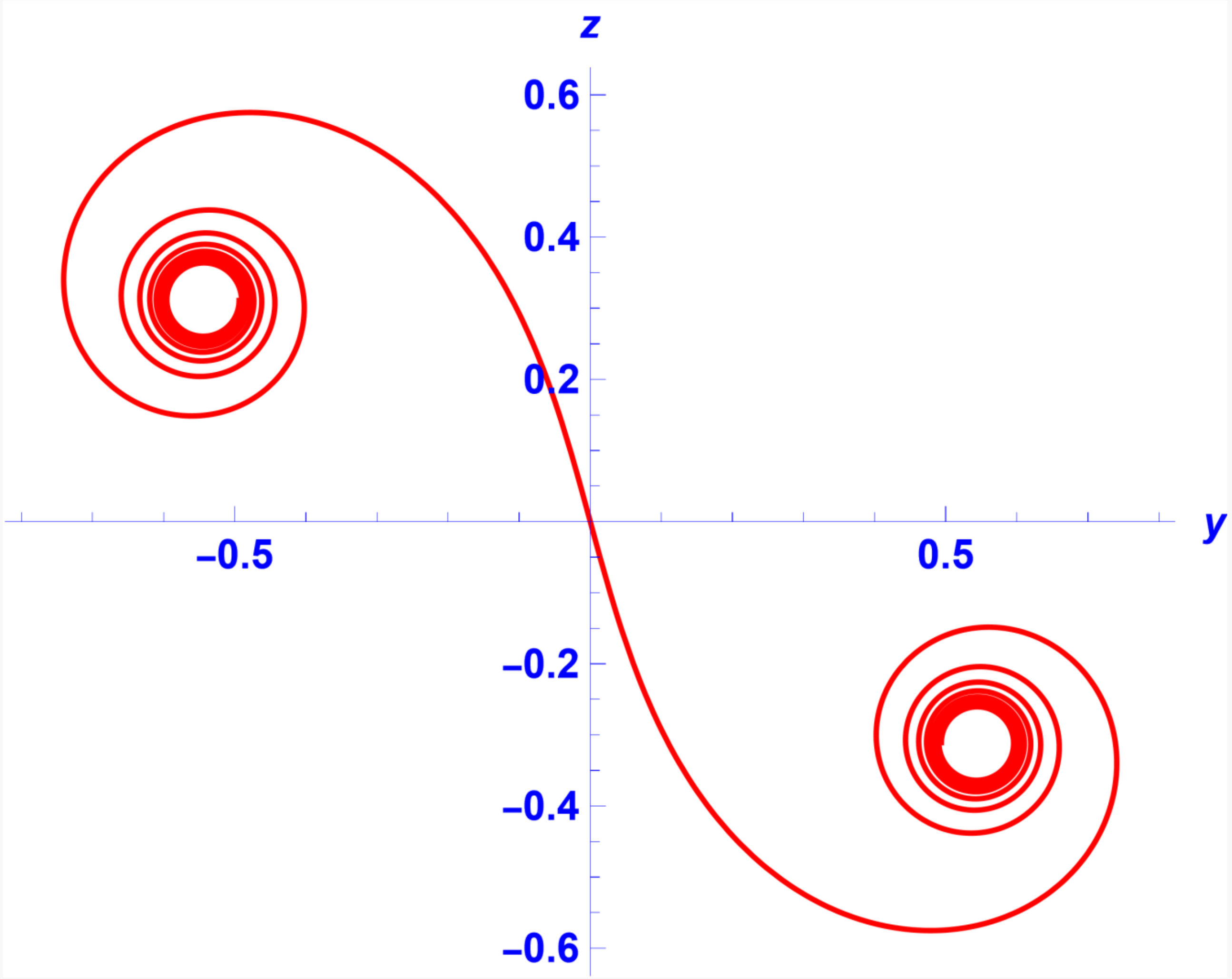
The projection of the motion of a classical particle with the same velocity the Weyl one, on the x-y plane. The settings are the same as in the previous figure.



The trajectory of a classical particle with the same velocity as the Weyl one for $\varphi(t)=\pi/2$ and $\theta(t)=20t-2t^2$, corresponding to a constant electric field $\mathbf{E}=(2/q)\mathbf{i}$, applied for $t\in[0,10]$



Thus, the localization of Weyl particles can be fully controlled, and modified, using simple electric fields, perpendicular to the direction of motion of the particles.

Furthermore, Weyl particles can be localized and delocalized at exceptionally high speeds. For example, they can be localized to a region of diameter 100 nm in a time interval well below 1 ps, for an electric field equal to 10V/mm.

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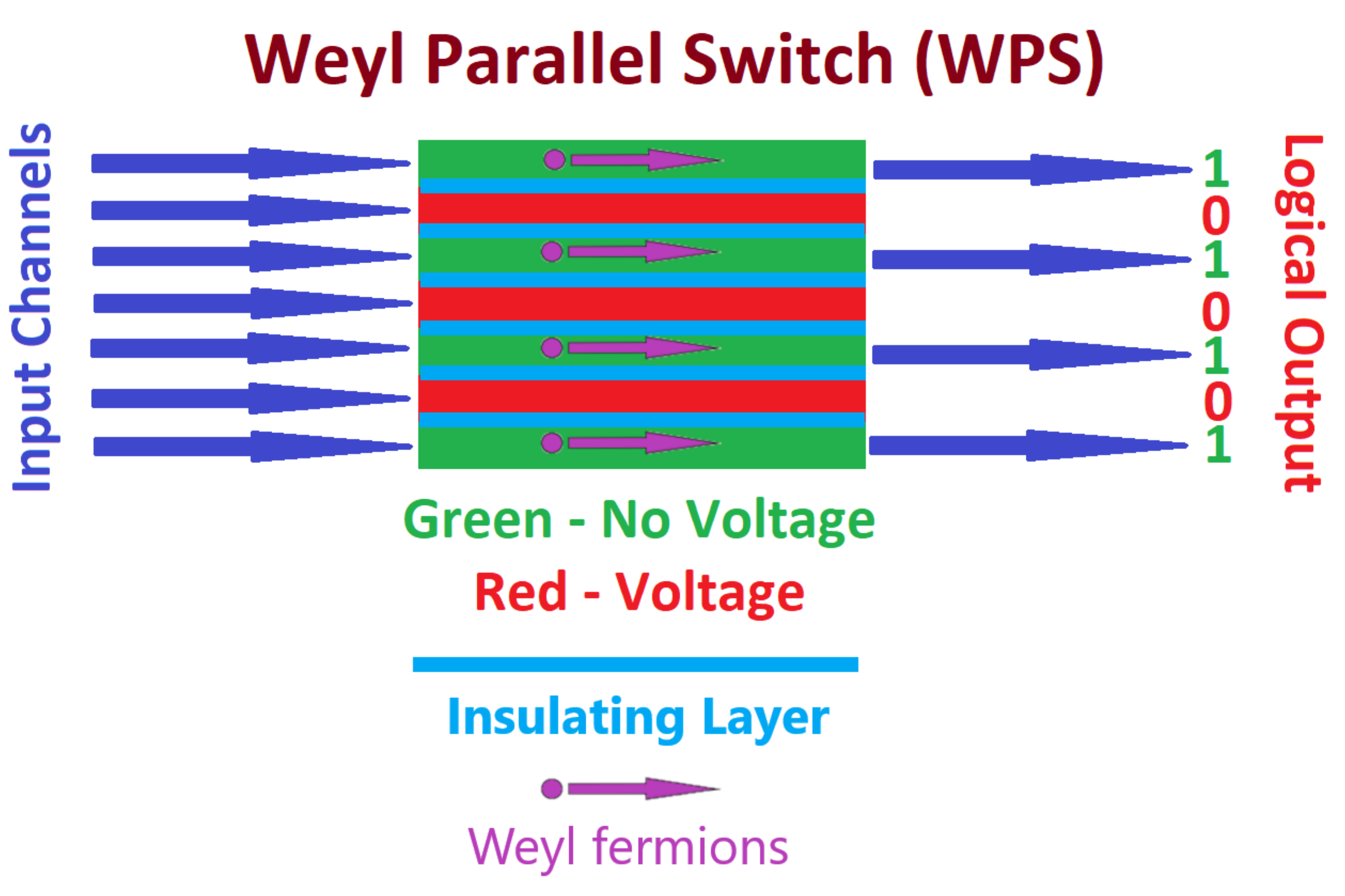
Based on these results we have proposed a novel device for controlling the flow of information at a rate of up to 100 Petabits per second using Weyl fermions, which can also be used as an electric field sensor



G. N. Tsigaridas, A. I. Kechriniotis, C. A. Tsonos and K. K. Delibasis, A proposed device for controlling the flow of information based on Weyl fermions, arXiv:2307.06489 [quant-ph], Sensors 24 (2024) 3361, DOI: 10.3390/s24113361

$$r(t) = \frac{r_0}{1 \pm \left(2qr_0 |\mathbf{E}|/\hbar\right)t}$$

$$\Delta t = \frac{\hbar}{2qr_0 |\mathbf{E}|}$$

Schematic diagram of a device for controlling the flow of information based on Weyl particles.

Controlling the spatial distribution of Weyl particles using appropriate magnetic fields

In the same article, we have also shown that it is possible to fully control the transverse spatial distribution, f(x,y), of Weyl particles using appropriate magnetic fields along the direction of motion of the particles, which can be used to guide Weyl fermions through the proposed device.

$$\mathbf{B} = -\frac{1}{q}\frac{1}{f^2}\left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 - f\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)\right]\mathbf{k}$$

General form of degenerate solutions for massless Dirac particles

In our effort to find general forms of degenerate solutions to the Dirac equation, we have found that all spinors of the form

$$\Psi_p = \left(c_1 u_\uparrow + c_2 u_\downarrow\right)\exp\left[ih\left(x,y,z,t\right)\right],\ \Psi_a = \left(c_1 v_\uparrow + c_2 v_\downarrow\right)\exp\left[ih\left(x,y,z,t\right)\right]$$

are degenerate corresponding to massless particles or antiparticles propagating along a direction in space defined by the angles (θ, ϕ) in spherical coordinates.

Here, c_1, c_2 are arbitrary complex constants, *h* is an arbitrary real functions of the spatial coordinates and time, and $u_\uparrow, u_\downarrow, v_\uparrow, v_\downarrow$, are the eigenvectors describing the spin state of the particles or antiparticles.

Electromagnetic 4-potentials corresponding to these solutions

The electromagnetic 4-potentials corresponding to these spinors are of the form

$$b_\mu = a_\mu + s\kappa_\mu$$

where

$$\left(a_0, a_1, a_2, a_3\right) = \left(\frac{\partial h}{\partial t}, \frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}, \frac{\partial h}{\partial z}\right)$$

$$\left(\kappa_0, \kappa_1, \kappa_2, \kappa_3\right) = \left(1, -\sin\theta\cos\varphi, -\sin\theta\sin\varphi, -\cos\theta\right)$$

Here, *s* is an arbitrary real function of the spatial coordinates and time.

Electromagnetic fields corresponding to these solutions

The electromagnetic fields corresponding to these 4-potentials are

$$\mathbf{E} = -\nabla U - \frac{\partial \mathbf{A}}{\partial t} = -\left(\sin\theta\cos\varphi \frac{\partial s_q}{\partial t} + \frac{\partial s_q}{\partial x} \right)\mathbf{i}$$

$$-\left(\sin\theta\sin\varphi \frac{\partial s_q}{\partial t} + \frac{\partial s_q}{\partial y} \right)\mathbf{j} - \left(\cos\theta \frac{\partial s_q}{\partial t} + \frac{\partial s_q}{\partial z} \right)\mathbf{k}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \left(-\sin\theta\sin\varphi \frac{\partial s_q}{\partial z} + \cos\theta \frac{\partial s_q}{\partial y} \right)\mathbf{i}$$

$$+\left(\sin\theta\cos\varphi \frac{\partial s_q}{\partial z} - \cos\theta \frac{\partial s_q}{\partial x} \right)\mathbf{j} + \sin\theta \left(-\cos\varphi \frac{\partial s_q}{\partial y} + \sin\varphi \frac{\partial s_q}{\partial x} \right)\mathbf{k}$$

Here $s_q = \frac{s}{q}$, where q is the electric charge of the particles.

A simplified form of these fields for particles moving along the +z-direction

Since the choice of the coordinate system is arbitrary, the direction of motion of the particles can be set to correspond to the +z-direction without loss of generality. In this case $\theta=\varphi=0$ and the electromagnetic fields take the simpler form

$$\mathbf{E} = -\frac{\partial s_q}{\partial x}\mathbf{i} - \frac{\partial s_q}{\partial y}\mathbf{j} - \left(\frac{\partial s_q}{\partial t} + \frac{\partial s_q}{\partial z}\right)\mathbf{k}$$

$$\mathbf{B} = \frac{\partial s_q}{\partial y}\mathbf{i} - \frac{\partial s_q}{\partial x}\mathbf{j}$$

An interesting remark regarding these solutions

Setting $h = E(z\text{-}t)$ in the case of particles or $h = -E(z\text{-}t)$ in the case of antiparticles, where E is an arbitrary real constant corresponding to the energy of the particles (or antiparticles), the degenerate spinors take the familiar form

$$\Psi_p = \left(c_1 u_\uparrow + c_2 u_\downarrow\right)\exp\left[iE\left(z-t\right)\right], \Psi_a = \left(c_1 v_\uparrow + c_2 v_\downarrow\right)\exp\left[-iE\left(z-t\right)\right]$$

corresponding to free massless particles or antiparticles moving along the +z-direction with energy E.

An example regarding electromagnetic waves

Setting

$$s_q = -E_{W1}\cos\left[k_W\left(z-t\right)+\delta_{W1}\right]x - E_{W2}\cos\left[k_W\left(z-t\right)+\delta_{W2}\right]y$$

where $E_{w1}, E_{w2}, \delta_{w1}, \delta_{w2}, k_w$ are real constants, the additional electromagnetic fields take the form

$$\mathbf{E}_s = E_{W1}\cos\left[k_W\left(z-t\right)+\delta_{W1}\right]\mathbf{i} + E_{W2}\cos\left[k_W\left(z-t\right)+\delta_{W2}\right]\mathbf{j}$$

$$\mathbf{B}_s = -E_{W2}\cos\left[k_W\left(z-t\right)+\delta_{W2}\right]\mathbf{i} + E_{W1}\cos\left[k_W\left(z-t\right)+\delta_{W1}\right]\mathbf{j}$$

corresponding to a plane electromagnetic wave, e.g., a laser beam, of arbitrary polarization, propagating along the +z – direction.

An example regarding electromagnetic waves (2)

Thus, the state of massless Dirac particles in degenerate states will not be affected in the presence of a plane electromagnetic wave, e.g., a laser beam, of arbitrary polarization, propagating along the direction of motion of the particles.

On the transition from degenerate to non-degenerate solutions as the particles acquire mass

Another very interesting aspect of degeneracy as defined in [1], is the study of the transition from a non-degenerate state corresponding to massive particles to a degenerate one corresponding to massless particles.

Specifically, it has been found that in the case of massive particles, the spinors

$$\Psi_p = \left(c_1 u_\uparrow + c_2 u_\downarrow\right)\exp\left[ih\left(x,y,z,t\right)\right],\ \Psi_a = \left(c_1 v_\uparrow + c_2 v_\downarrow\right)\exp\left[ih\left(x,y,z,t\right)\right]$$

are still solutions to the Dirac equation for the 4-potentials

$$\left(a_0,a_1,a_2,a_3\right) = \left(\frac{\partial h}{\partial t},\frac{\partial h}{\partial x},\frac{\partial h}{\partial y},\frac{\partial h}{\partial z}\right)$$

On the transition from degenerate to non-degenerate solutions as the particles acquire mass(2)

However, these spinors are no longer degenerate, and consequently they are not solutions to the Dirac equation for the more general 4-potentials

$$b_\mu = a_\mu + s\kappa_\mu$$

Indeed, substituting these spinors into the massless Dirac equation for the above 4-potentials, it takes the form

$$i\gamma^\mu \partial_\mu \Psi_p + b_\mu \gamma^\mu \Psi_p = \left(1 - \sqrt{\frac{1-e}{1+e}}\right) s \Psi_p{}^*$$

in the case of particles, and

$$i\gamma^\mu \partial_\mu \Psi_a + b_\mu \gamma^\mu \Psi_a = -\left(1 - \sqrt{\frac{1-e}{1+e}}\right) s \Psi_a{}^*$$

in the case of antiparticles.

Here $\Psi_p{}^*, \Psi_a{}^*$ are the unperturbed spinors for particles and antiparticles respectively, and e is the ratio of the rest energy of the particles to their total energy ($e=m/E$ in natural units)

On the transition from degenerate to non-degenerate solutions as the particles acquire mass(3)

In the case that the rest energy of the particles is much smaller than their total energy, or $e<<1$, the factor

$$\left(1-\sqrt{\frac{1-e}{1+e}}\right)s$$

takes the simple form *e s*. Thus, as the mass of the particles increases, the function *s* should be restricted to smaller values, suppressing the effect of degeneracy.

On the other hand, as the mass of the particles decreases, the function *s* is allowed to take larger values, and the degeneracy becomes more evident.

Finally, as the mass of the particles tends to zero, there is no restriction on the values of the function *s*, and the theory of degeneracy becomes fully applicable.

Important remark

The parameter *e,* which is the ratio of the rest energy of the particles to their total energy, *$e=mc^2/E$, becomes also negligible if the total energy of the particles is much higher than their rest energy, $E>>mc^2$.*

Thus, the theory of degeneracy is also expected to be valid for high energy particles.

Furthermore, the higher the total energy of the particles compared to their rest energy, the more evident the effects of degeneracy are expected to become.

Degenerate solutions for massive particles (1)

All the previous results consider degenerate solutions for free Weyl and massless Dirac particles. However, an interesting question is the following:

Are there degenerate solutions for massive particles, and if yes, what is their physical interpretation?

Degenerate solutions for massive particles (2)

Indeed, it has been shown that all spinors of the form

$$\Psi = c_1 \exp\left(if \cos\xi\right)\exp\left[-\frac{m}{\sin^2\xi}\left(-z+t\cos\xi\right)\right]\begin{pmatrix} i\sin\xi \\ -i-\cos\xi \\ \sin\xi \\ 1+i\cos\xi \end{pmatrix}$$

corresponding to particles with arbitrary mass *m* are degenerate.

The electromagnetic 4-potentials connected to the above spinors are the following:

$$\left(a_0, a_1, a_2, a_3\right) = \cos\xi\left(\frac{\partial f}{\partial t}, \frac{\partial f}{\partial x} - \frac{m}{\sin\xi}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

Here, c_1 is an arbitrary complex constant, $\xi \neq n\pi, n \in \mathbb{Z}$ is a real parameter and f is an arbitrary real function of the spatial coordinates and time.

Degenerate solutions for massive particles (3)

Further, according to Theorem 5.4 in [1], these spinors will also be solutions to the Dirac equation for an infinite number of 4-potentials, given by the formula

$$b_\mu = a_\mu + s\kappa_\mu$$

where

$$(\kappa_0, \kappa_1, \kappa_2, \kappa_3) = (1, 0, \sin\xi, -\cos\xi)$$

and *s* is an arbitrary real function of the spatial coordinates and time.

On the physical interpretation of these results in the framework of quantum tunneling (1)

Setting $\xi = n\pi + \pi/2, n \in \mathbb{Z}$ the degenerate spinors take the simpler form

$$\Psi = c_1 \exp(m\,z) \begin{pmatrix} \pm i \\ -i \\ \pm 1 \\ 1 \end{pmatrix}$$

which are also solutions to the one dimensional Dirac equation

$$i\gamma^3 \partial_z \Psi - m\Psi = 0$$

for zero 4-potential.

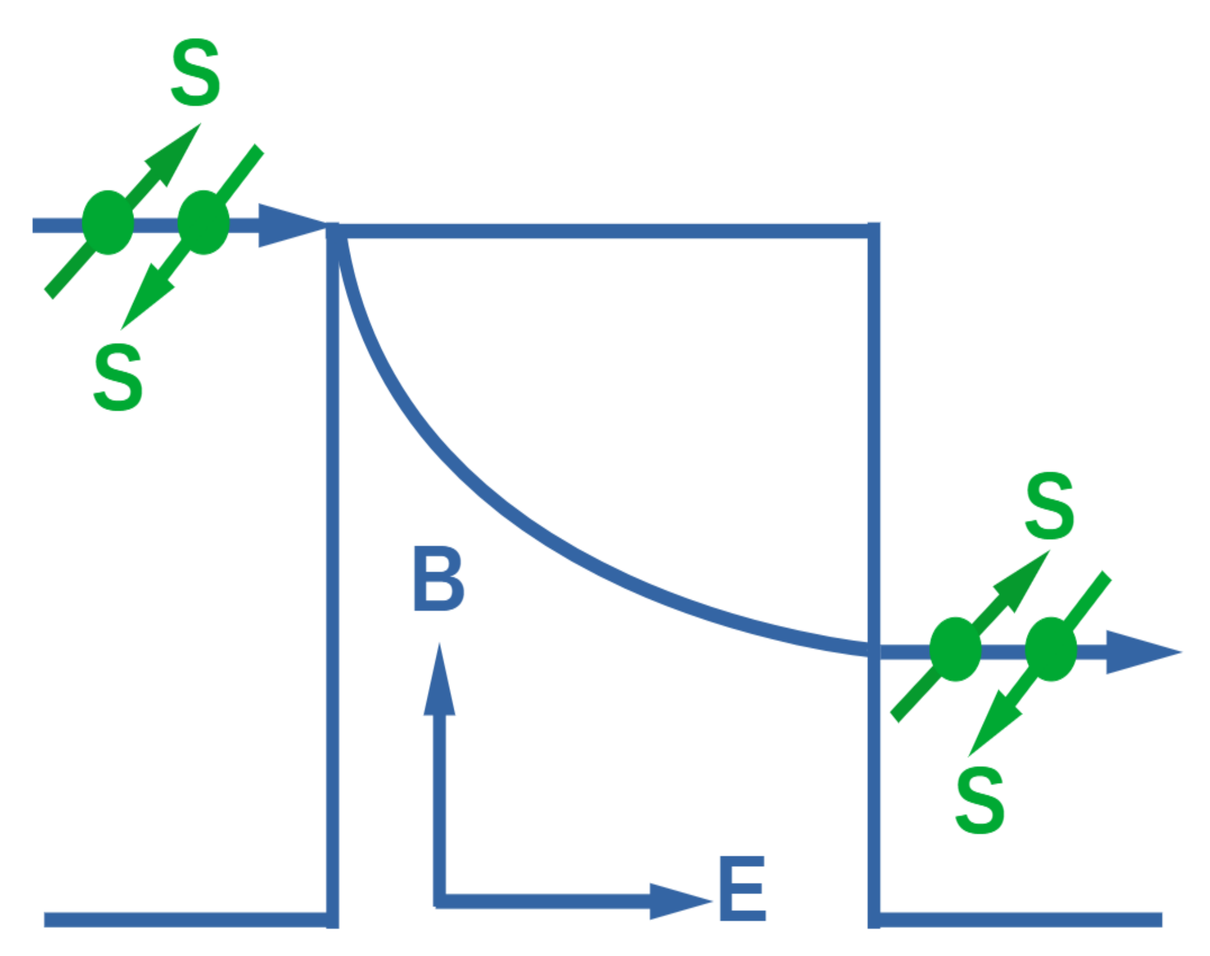
On the physical interpretation of these results in the framework of quantum tunneling (2)

In general, it can be shown that all spinors of the form

$$\Psi_p = c_+ \exp(-m\,z) \begin{pmatrix} 1 \\ 1 \\ i \\ -i \end{pmatrix} + c_- \exp(m\,z) \begin{pmatrix} -1 \\ 1 \\ i \\ i \end{pmatrix}, \quad \Psi_a = c_+ \exp(m\,z) \begin{pmatrix} i \\ -i \\ 1 \\ 1 \end{pmatrix} + c_- \exp(-m\,z) \begin{pmatrix} i \\ i \\ -1 \\ 1 \end{pmatrix}$$

are degenerate solutions to the Dirac equation for zero 4-potential corresponding to single particles with spin perpendicular to the z-axis or pairs of massive particles or antiparticles moving along the z-axis in a potential barrier with height equal to the energy of the particles. Further, the spin of the particles is opposite to each other, so that the projection of the total spin on the z-axis is equal to zero.

On the physical interpretation of these results in the framework of quantum tunneling (3)



The electromagnetic fields corresponding to these degenerate solutions

According to Theorem 5.4 in [1], these spinors will also be solutions to the Dirac equation for the 4-potentials

$$(b_0, b_1, b_2, b_3) = s(1,0,1,0)$$

corresponding to the following electromagnetic fields

$$\mathbf{E} = -\frac{\partial s_q}{\partial x}\mathbf{i} + \left(\frac{\partial s_q}{\partial t} - \frac{\partial s_q}{\partial y}\right)\mathbf{j} - \frac{\partial s_q}{\partial z}\mathbf{k}, \quad \mathbf{B} = \frac{\partial s_q}{\partial z}\mathbf{i} - \frac{\partial s_q}{\partial x}\mathbf{k}$$

Here, s is an arbitrary real function of the spatial coordinates and time and $s_q = \frac{s}{q}$, where q is the electric charge of the particles.

A specific example regarding electromagnetic waves (1)

Setting

$$s_q = -E_{W1}\cos\left[k_W\left(y+t\right)+\delta_{W1}\right]x - E_{W2}\left[k_W\left(y+t\right)+\delta_{W2}\right]z$$

the electromagnetic fields take the form

$$\mathbf{E}_W = E_{W1}\cos\left[k_W\left(y+t\right)+\delta_{W1}\right]\mathbf{i} + E_{W2}\cos\left[k_W\left(y+t\right)+\delta_{W2}\right]\mathbf{k}$$

$$\mathbf{B}_W = -E_{W2}\cos\left[k_W\left(y+t\right)+\delta_{W2}\right]\mathbf{i} + E_{W1}\cos\left[k_W\left(y+t\right)+\delta_{W1}\right]\mathbf{k}$$

corresponding to a plane electromagnetic wave, of arbitrary polarization, propagating along the y-direction.

Here, $E_{w1}, E_{w2}, \delta_{w1}, \delta_{w2}$ are arbitrary real constants corresponding to the amplitude and phase of the x and z component of the electric field of the wave respectively, and k_w is a real parameter corresponding to the wavenumber.

A specific example regarding electromagnetic waves (2)

Further, since the choice of the coordinate system is arbitrary, the y-direction can be set to correspond to any desired direction in space, perpendicular to the direction of motion of the particles.

Thus, the state of the particles inside the potential barrier, and consequently the transmittance though the barrier, will not be affected by the presence of a plane electromagnetic wave, e. g. a laser beam, with arbitrary polarization, propagating along any direction perpendicular to the direction of motion of the particles.

On the practical applications of these results

The property of particles described by degenerate spinors to be in the same state under a wide variety of different electromagnetic fields, provides the opportunity to apply certain combinations of electric and magnetic fields in order to manipulate the motion of the particles in free space, without affecting their state inside the potential barrier, and consequently, the transmittance through the barrier.

More details can be found in [2].

Nearly degenerate solutions

An interesting question is what happens in the case of small deviations from the conditions for degeneracy.

For example, what happens if the energy of the particles is not exactly equal to the height of the potential barrier?

Nearly degenerate solutions (2)

We have found that, in this case, the Dirac equation takes the form

$$i\gamma^{\mu}\partial_{\mu}\Psi_{d,p}(+z)+b_{\mu}\gamma^{\mu}\Psi_{d,p}-m\Psi_{d,p}(+z)=c_0 em\exp(-m\,z)\begin{pmatrix}-1\\-1\\i\\-i\end{pmatrix}+c_0 es\exp(-m\,z)\begin{pmatrix}1\\1\\i\\-i\end{pmatrix}$$

where $\Psi_{d,p}$ is the degenerate Dirac spinor and $e=|E-V_0|/m<<1$ is a small factor describing the deviation of the energy of the particles from the exact condition for degeneracy ($E=V_0$).

Nearly degenerate solutions (3)

Consequently, the terms

$$c_0 em \exp(-m\,z)\begin{pmatrix}-1\\-1\\i\\-i\end{pmatrix}+c_0 es \exp(-m\,z)\begin{pmatrix}1\\1\\i\\-i\end{pmatrix}$$

can be ignored, provided that e << 1 and e s << m.

The physical interpretation of these conditions is that the deviation of the energy of the particles from the exact condition for degeneracy ($E=V_0$) as well as the energy corresponding to the electromagnetic potential induced by the function s must be much smaller than the rest energy of the particles.

Nearly degenerate solutions (4)

Thus, the results obtained regarding the properties of degenerate solutions, will still be valid in the case of small deviations from the condition for degeneracy ($E=V_0$), provided that the arbitrary function s takes sufficiently small values, so that the energy corresponding to the electromagnetic potential induced by this function is much smaller than the rest energy of the particles.

In addition, the larger the deviation from the exact degenerate solution is, the smaller the values of the function s should be, as it is expected from a physical point of view.

Degenerate wave-like solutions to the Dirac equation for massive particles

It can be verified that all spinors of the following form

$$\Psi = c_1 \exp(ih) \begin{pmatrix} \cos\alpha \\ \sin\alpha \exp(id) \\ \cos\beta \\ \sin\beta \exp(id) \end{pmatrix}$$

where c_1 is an arbitrary complex constant, α , β are real constants, h is an arbitrary real function of the spatial coordinates and time and

$$d = \frac{4m\left[t - z\cos(\alpha+\beta)\right]}{\cos(2\alpha) - \cos(2\beta)}$$

are degenerate solutions to the Dirac equation.

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Electromagnetic 4-potentials corresponding to these solutions

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -\dfrac{\tan(\alpha+\beta)\left(\left(\sin(2\alpha)-\sin(2\beta)\right)\dfrac{\partial h}{\partial t}+m\left(\sin(2\alpha)+\sin(2\beta)\right)\right)}{\cos(2\alpha)-\cos(2\beta)} \\ -2m\cos\alpha\cos\beta\csc(\alpha-\beta)\sec(\alpha+\beta)\cos d+\dfrac{\partial h}{\partial x} \\ -2m\cos\alpha\cos\beta\csc(\alpha-\beta)\sec(\alpha+\beta)\sin d+\dfrac{\partial h}{\partial y} \\ \dfrac{\partial h}{\partial z} \end{pmatrix}$$

$$\alpha\pm\beta\neq n\pi,\alpha+\beta\neq n\pi+\pi/2,n\in\mathbb{Z}$$

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Electromagnetic fields corresponding to these solutions

$$\mathbf{E} = -\nabla U - \frac{\partial \mathbf{A}}{\partial t}$$
$$= 4m^2 \cos\alpha \cos\beta \csc^2(\alpha-\beta)\csc(\alpha+\beta)\sec(\alpha+\beta)(-\sin d\ \mathbf{i} + \cos d\ \mathbf{j})$$

$$\mathbf{B} = \nabla\times\mathbf{A} = -4m^2 \cos\alpha \cos\beta \csc^2(\alpha-\beta)\csc(\alpha+\beta)(\cos d\ \mathbf{i} + \sin d\ \mathbf{j})$$

$$\mathbf{S} = \frac{1}{4\pi}\mathbf{E}\times\mathbf{B} = \frac{4m^4}{\pi}\cos^2\alpha\cos^2\beta\csc^4(\alpha-\beta)\csc^2(\alpha+\beta)\sec(\alpha+\beta)\mathbf{k}$$

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Additional electromagnetic 4-potentials and fields corresponding to these solutions

$$b_\mu = a_\mu + s\kappa_\mu$$

$$(\kappa_0, \kappa_1, \kappa_2, \kappa_3) = \left(1, -\frac{\Psi^T \gamma^0 \gamma^1 \gamma^2 \Psi}{\Psi^T \gamma^2 \Psi}, -\frac{\Psi^T \gamma^0 \Psi}{\Psi^T \gamma^2 \Psi}, \frac{\Psi^T \gamma^0 \gamma^2 \gamma^3 \Psi}{\Psi^T \gamma^2 \Psi}\right)$$

$$= \left(1, -\sin(\alpha+\beta)\cos d, -\sin(\alpha+\beta)\sin d, -\cos(\alpha+\beta)\right)$$

$$\mathbf{E} = -\nabla U - \frac{\partial \mathbf{A}}{\partial t} \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

$$U = \frac{1}{q} b_0 \qquad \mathbf{A} = -\frac{1}{q}\left(b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}\right)$$

As usually, *s* is an arbitrary real function of the spatial coordinates and time

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Interesting remark (1)

If the following conditions hold

$$\frac{\partial h}{\partial x}=\frac{\partial h}{\partial y}=\frac{\partial h}{\partial z}=0$$

$$\alpha=n\pi+\frac{\pi}{2}\text{ or }\beta=n\pi+\frac{\pi}{2},\quad n\in\mathbb{Z}$$

$$\frac{\partial h}{\partial t}=-m\frac{\sin(2\alpha)+\sin(2\beta)}{\sin(2\alpha)-\sin(2\beta)}$$

the electromagnetic 4-potentials and fields become zero.

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Interesting remark (2)

The phase velocity of the electromagnetic waves, defined as

$$\upsilon_{ph} = \frac{\omega_d}{k_d} = \sec(\alpha + \beta)$$

$$\omega_d = \frac{4m}{\cos(2\alpha) - \cos(2\beta)} \qquad k_d = \frac{4m\cos(\alpha + \beta)}{\cos(2\alpha) - \cos(2\beta)}$$

is higher than the speed of light in vacuum. However, this does not violate the special theory of relativity since a sinusoidal wave with a unique frequency does not transmit any information. Indeed, the phase velocity of an electromagnetic wave when traveling through a medium can exceed the speed of light in vacuum, as it happens in most glasses at X-ray frequencies and in unmagnetized plasmas.

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Interesting remark (3)

The frequency of the oscillation is proportional to the mass of the particles

$$f_d\left(\text{S.I.}\right)=\frac{\omega_d\left(\text{S.I.}\right)}{2\pi}=\frac{4mc^2}{h}\frac{1}{\cos\left(2\alpha\right)-\cos\left(2\beta\right)}$$

and takes particularly high values. For example, in the case of electrons

$$f_d\left(\text{S.I.}\right)=\frac{4.95\times 10^{20}}{\cos\left(2\alpha\right)-\cos\left(2\beta\right)}Hz$$

corresponding to photons with energy higher than 2.05 MeV, in the region of Gamma/X-rays. Obviously, in the case of heavier particles, e.g., protons , the frequency of the oscillation and the energy of the photons take much higher values.

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Interesting remark (4)

The expected values of the projections of the spin of the particles along the x, y, and z-axes, defined through the following formulae

$$S_x = \frac{i}{2}\Psi^\dagger\gamma^2\gamma^3\Psi = \frac{|c_1|^2}{2}\left(\sin(2\alpha)+\sin(2\beta)\right)\cos d$$

$$S_y = \frac{i}{2}\Psi^\dagger\gamma^3\gamma^1\Psi = \frac{|c_1|^2}{2}\left(\sin(2\alpha)+\sin(2\beta)\right)\sin d$$

$$S_z = \frac{i}{2}\Psi^\dagger\gamma^1\gamma^2\Psi = \frac{|c_1|^2}{2}\left(\cos(2\alpha)+\cos(2\beta)\right)$$

are synchronized with the magnetic component of the electromagnetic fields corresponding to these solutions.

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A general method for obtaining degenerate solutions to the Dirac and Weyl equations

In the article

G. N. Tsigaridas, A. I. Kechriniotis, C. A. Tsonos and K. K. Delibasis, A general method for obtaining degenerate solutions to the Dirac and Weyl equations and a discussion on the experimental detection of degenerate states, arXiv:2210.02003 [quant-ph], Ann. Phys. (Berlin) 2200647 (2023) DOI: 10.1002/andp.202200647

we have shown that all spinors of the following form

$$\tilde{\Psi} = \exp\left(i\int \tilde{f}_{1I}(s_0,s_1)ds_1 + \tilde{f}_{2R}(s_0)(s_2+s_3)+i\tilde{f}_{2I}(s_0)(s_2-s_3)\right)$$

$$\times g(s_0)\exp\left(-\frac{m^2\cos^2\varphi}{k}s_2\right)\exp(ks_3)$$

$$\times\left(i\frac{m(1+\sin\varphi)}{k}\begin{pmatrix}\cos\varphi\\1-\sin\varphi\\\cos\varphi\\1-\sin\varphi\end{pmatrix}+\begin{pmatrix}-\cos\varphi\\1+\sin\varphi\\\cos\varphi\\-1-\sin\varphi\end{pmatrix}\right)$$

m is the mass of the particle

$g(s_0)$ is an arbitrary complex function of s_0

k is an arbitrary complex constant

$\varphi \neq n\pi+\pi/2,\ \ n\in\mathbb{Z}$ is a real constant

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Electromagnetic 4-potentials corresponding to these solutions

are degenerate solutions to the Dirac equation for the following 4-potentials:

$$a_0 = h$$

$$a_1 = -h\cos\varphi + f_{1I}\sec\varphi + f_{2I}\tan\varphi$$

$$a_2 = f_{2R}\sec\varphi$$

$$a_3 = -h\sin\varphi - f_{2I}$$

where f_{1I}, f_{2R}, f_{2I}, h are arbitrary real functions of the spatial coordinates and time.

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Coordinate transformation

The functions $\tilde{f}_{1I}(s_0, s_1), \tilde{f}_{2R}(s_0), \tilde{f}_{2I}(s_0)$ are related to f_{1I}, f_{2R}, f_{2I} through the following transformation of the coordinates

$$\begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_0 \end{pmatrix} = \begin{pmatrix} \sec\varphi & 0 & 0 & 0 \\ \frac{1}{2}\tan\varphi & \frac{i}{2}\sec\varphi & -\frac{1}{2} & 0 \\ -\frac{1}{2}\tan\varphi & \frac{i}{2}\sec\varphi & \frac{1}{2} & 0 \\ -\cos\varphi & 0 & -\sin\varphi & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

Thus, for any combination of the arbitrary functions $\tilde{f}_{1I}(s_0, s_1), \tilde{f}_{2R}(s_0), \tilde{f}_{2I}(s_0)$ one can automatically construct a degenerate solution to the Dirac equation corresponding to an infinite number of real 4-potentials, given by the equations in the previous slight.

Important remarks

- Massive particles described by those spinors should be localized, both in space and time, because otherwise the solution would be divergent
- All the information regarding the 4-potentials is incorporated into the phase of the spinors
- The expected values of the projections of the spin of the particles along the x-, y-, and z-axes are functions of the mass of the particles and the spatial and temporal coordinates
- However, in the special case that $k = m\cos\varphi$ the expected values of the projections of the spin of the particles along the x-, y-, and z-axes become all equal to zero.

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Degenerate solutions for massless Dirac particles

In the case of massless particles, the degenerate solutions take the following form:

$$\tilde{\Psi} = \exp\left(i\int \tilde{f}_{1I}\left(s_0,s_1\right)ds_1 + \tilde{f}_{2R}\left(s_0\right)\left(s_2+s_3\right)+i\tilde{f}_{2I}\left(s_0\right)\left(s_2-s_3\right)\right)$$

$$\times\left(\tilde{W}_T\left(s_0,s_2\right)\begin{pmatrix}\cos\varphi\\ 1-\sin\varphi\\ \cos\varphi\\ 1-\sin\varphi\end{pmatrix}+\tilde{W}_R\left(s_0,s_3\right)\begin{pmatrix}-\cos\varphi\\ 1+\sin\varphi\\ \cos\varphi\\ -1-\sin\varphi\end{pmatrix}\right)$$

where $\tilde{W}_T\left(s_0,s_2\right),\tilde{W}_R\left(s_0,s_3\right)$ are arbitrary complex functions of the coordinates s_0,s_2 and $s_0,s_3,$ respectively.

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Important remarks

- The localization of the spinors is no longer required, and consequently massless Dirac particles can move freely throughout space and time.
- All the information regarding the 4-potentials is incorporated into the phase of the spinors, as in the case of massive Dirac particles
- The expected values of the projections of the spin of the particles along the x, y, and z-axes are all constants

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Degenerate solution for Weyl particles

In the case that $\tilde{W}_R(s_0, s_3) = 0$ or $\tilde{W}_T(s_0, s_2) = 0$,

the degenerate spinors corresponding to massless particles take the simpler form

$\tilde{\Psi} = (\tilde{\psi}_T, \tilde{\psi}_T)^T$ and $\tilde{\Psi} = (\tilde{\psi}_R, -\tilde{\psi}_R)^T$ respectively, where

$$\tilde{\psi}_T = \exp\left(i\int \tilde{f}_{1I}(s_0, s_1)ds_1 + \tilde{f}_{2R}(s_0)(s_2 + s_3) + i\tilde{f}_{2I}(s_0)(s_2 - s_3)\right)\tilde{W}_T(s_0, s_2)\begin{pmatrix} \cos\varphi \\ 1 - \sin\varphi \end{pmatrix}$$

is solution to the Weyl equation

$$i\sigma^\mu \partial_\mu \Psi + a_\mu \sigma^\mu \Psi = 0$$

corresponding to particles with positive helicity and

$$\tilde{\psi}_R = \exp\left(i\int \tilde{f}_{1I}(s_0, s_1)ds_1 + \tilde{f}_{2R}(s_0)(s_2 + s_3) + i\tilde{f}_{2I}(s_0)(s_2 - s_3)\right)\tilde{W}_R(s_0, s_3)\begin{pmatrix} -\cos\varphi \\ 1 + \sin\varphi \end{pmatrix}$$

is solution to the Weyl equation

$$i\sigma^\mu \partial_\mu \Psi - 2i\sigma^0 \partial_0 \Psi + a_\mu \sigma^\mu \Psi - 2a_0 \sigma^0 \Psi = 0$$

corresponding to particles with negative helicity.

Final remarks

- As in the case of massless Dirac particles, Weyl particles can move freely throughout space and time.
- The phase factor containing the information regarding the electromagnetic 4-potentials and fields is the same for massive Dirac, massless Dirac and Weyl particles.
- For any combination of the arbitrary real functions

$$\tilde{f}_{1I}\left(s_0, s_1\right), \tilde{f}_{2R}\left(s_0\right), \tilde{f}_{2I}\left(s_0\right)$$

one can automatically construct degenerate solutions for massive Dirac, massless Dirac and Weyl particles, corresponding to the same electromagnetic 4-potentials and fields.

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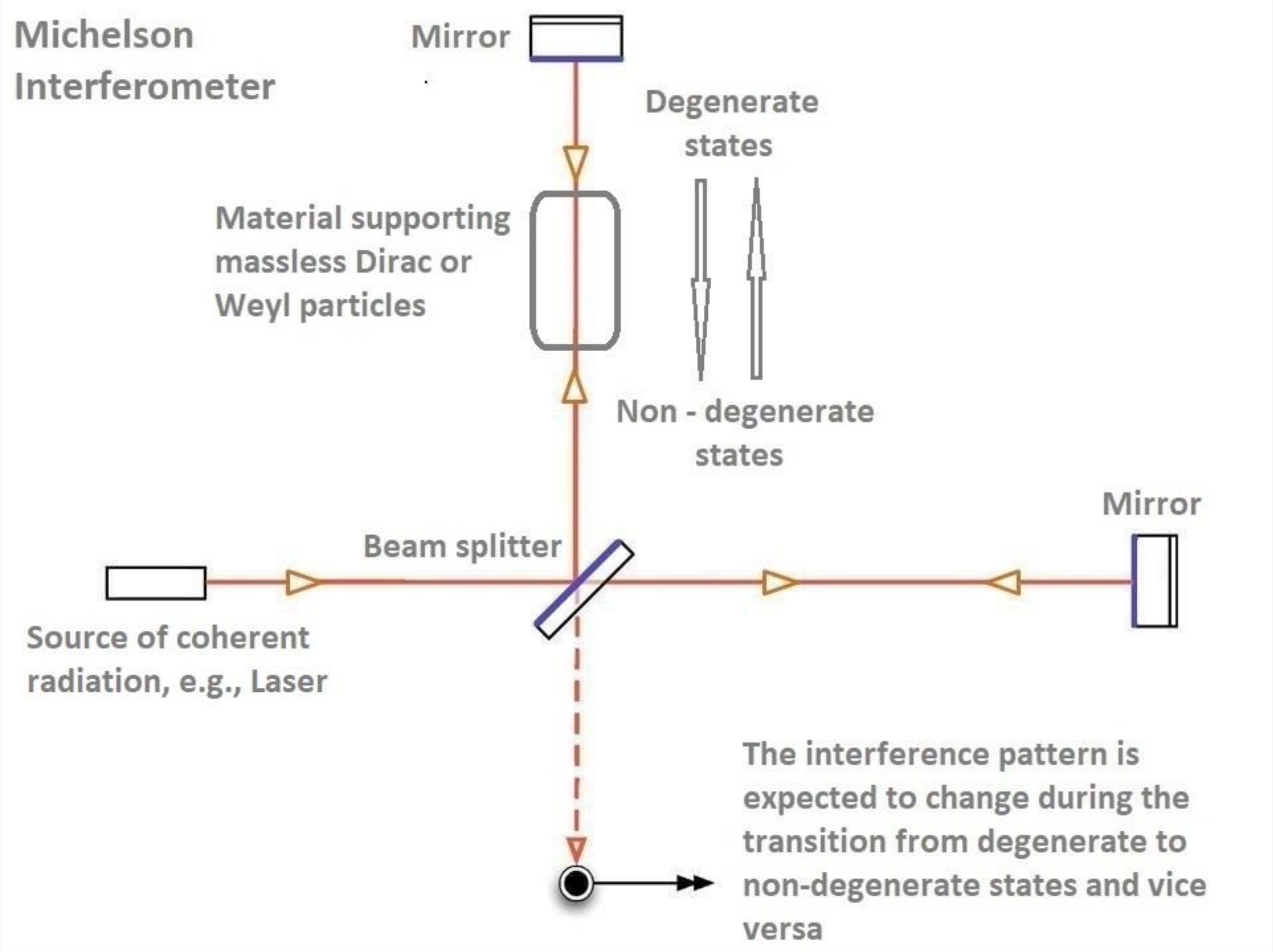
A proposed method for experimentally detecting the transition between degenerate and non – degenerate states



General remarks - Conclusions (1)

In conclusion, we have shown that all Weyl particles, and under certain conditions Dirac particles, interact with the electromagnetic fields in new, unexpected ways, being able to exist in the same quantum state under a wide variety of different electromagnetic fields, including both spatially constant fields and electromagnetic waves.

We have also shown that Weyl particles can exist in different states in zero electromagnetic field, either as free particles, or in localized states.

The localization, as well as the energy, of the particles can be fully controlled using simple electric fields, which can be easily realized in practice.

General remarks - Conclusions (2)

Based on these results we have proposed a quantum parallel switch based on Weyl fermions, which is expected to be able to control the flow of information at a rate of up to 100 Petabits per second, offering also several advantages over conventional electronics, as low power consumption and robustness against electromagnetic perturbations, rendering it ideal for a variety of applications in the fields of telecommunications, signal processing, quantum computing, etc.

We have also proposed a method for controlling the spatial distribution of Weyl particles using appropriate magnetic fields

General remarks - Conclusions (3)

We have provided degenerate solutions to the Dirac equation corresponding to massive particles in potential barriers.

We have also calculated the family of electromagnetic fields that should be applied in order to control the state of the particles in the region of the potential barrier, without affecting the state of the particles inside the potential barrier, and consequently the transmittance through the barrier.

General remarks - Conclusions (4)

We have provided several classes of degenerate solutions corresponding to massless Dirac and Weyl particles and we have calculated the electromagnetic 4-potentials and fields corresponding to these solutions.

It has been shown that the state of the particles will not be affected in the presence of an electric field, of arbitrary magnitude and time dependence, applied along their direction of motion, or a plane electromagnetic wave, e.g. a laser beam, of arbitrary polarization, propagating along the direction of motion of the particles.

The transition from degenerate to non-degenerate states as the particles acquire mass has also been studied.

General remarks - Conclusions (5)

We have provided a class of wavelike solutions to the Dirac equation for massive particles, where the spin of the particles is synchronized with the magnetic component of the electromagnetic fields corresponding to these solutions.

Finally, we have developed a general method for systematically obtaining degenerate solutions to the Dirac (both for massive and massless particles) and Weyl equations and we have proposed a method for experimentally detecting the transition between degenerate and non-degenerate states.

Thank you for your attention…